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LETTER TO THE EDITOR

The transverse Ising system with arbitrary spin

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Abstract. Transverse Ising models with arbitrary spin S are studied by use of the pair approximation with the discretized path-integral representation. The equation for the phase diagram is derived for an arbitrary number of nearest-neighbour spins z and critical lines are obtained numerically for a few values of the spin. It is easy to show that in the limit $z \rightarrow \infty$ the results for the infinite-range interaction model can be derived within the present formulation.

Theories of the quantum spin model have received much attention recently [1–8]. The transverse spin-$\frac{1}{2}$ Ising model is useful for the study of cooperative phenomena and phase transitions in many systems [9–16], and has been studied in the ordinary mean-field approximation (MFA) and an improved MFA which includes a fluctuation correction [5]. In two or more dimensions, the transverse Ising model (TIM) has a finite-temperature phase transition, which can be depressed to zero temperature, that is also critical at a critical value of the transverse field. The TIM therefore serves as a model of quantum-critical phenomena at zero temperature. Recently, a new method [7, 8], which combines the pair approximation with the discretized path-integral representation (DPIR) of a quantum spin-$\frac{1}{2}$ system, has been proposed. The results obtained are much better than that with the MFA.

However, as far as we know, up to now no studies have made contact with the quantum transverse spin-S Ising model. In this paper, we shall discuss transverse Ising models with arbitrary spin, in the pair approximation. An analytical expression for the phase diagram is derived. This equation is resolved numerically for a few values of the spin.

The Hamiltonian of the system is given by

$$H = -J \sum_{\langle ij \rangle} S_i^z S_j^z - \Gamma \sum_i S_i^x \tag{1}$$

where S_i^z and S_i^x are the quantum spin-S operators at site i, and Γ denotes a transverse field.

The one-body effect Hamiltonian in the pair approximation is given by

$$H_i = -\Gamma S_i^x - H^{\mathrm{ef}} S_i^z \tag{2}$$

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where H^{ef} is the one-body effective field. The corresponding one-body partition function becomes

$$Z_i(\beta, \Gamma, H^{ef}) = \mathrm{Tr}\ \exp(-\beta H_i). \tag{3}$$

The pair Hamiltonian is given by

$$H_{ij} = J S_i^z S_j^z - \Gamma\left(S_i^x + S_j^x\right) - h^{ef}\left(S_i^z + S_j^z\right) \tag{4}$$

where h^{ef} is the effective field, which is related to the effective field H^{ef} by $h^{ef} = H^{ef}(z-1)/z$. In order to circumvent the problem of non-commuting operators in the pair Hamiltonian, we will reformulate the Hamiltonian in DPIR to obtain the pair partition function [5, 7, 8]. In the DPIR, the quantal $(2S+1)$-state spin on each lattice site will be transformed into a P-component vector, and eventually P will be allowed to go to infinity. Each component is taken to be a classical spin variable, and the net effect is to represent the quantum uncertainty by creating many copies, or replicas, of the original variables. By means of the DPIR, the pair Hamiltonian can be broken up into a reference part involving only the single-site terms, and an interaction part. The corresponding free energy can be expressed in terms of the free energy of the reference part and a cumulant expansion. By taking the first cumulant, we obtain the expression

$$\begin{aligned}
&\ln Z_{ij}(\beta, \Gamma, h^{ef}) = \ln Z_i(\beta, \Gamma, h^{ef}) + \ln Z_j(\beta, \Gamma, h^{ef}) + \beta J M_i M_j \\
&M_i = \beta^{-1}\ \partial \ln Z_i(\beta, \Gamma, h^{ef})/\partial h^{ef}.
\end{aligned} \tag{5}$$

The free energy $f(h^{ef})$ per spin in the pair approximation is given by the expression

$$-\beta f(h^{ef}) = (1-z) \ln Z_i(\beta, \Gamma, H^{ef}) + (z/2) \ln Z_{ij}(\beta, \Gamma, h^{ef}). \tag{6}$$

Second-order transition lines can be obtained from the zero point of the coefficient of the second-order term in (6) when the equation is expanded in terms of h^{ef}. The final result reads

$$\chi(\beta, \Gamma) = 1/(z-1)J \qquad \chi(\beta, \Gamma) = \beta^{-1}\ \partial^2 \ln Z_i(\beta, \Gamma, h^{ef})/\partial (h^{ef})^2|_{h^{ef}=0}. \tag{7}$$

This determines the ferromagnetic transition temperature dependence on the transverse field Γ for arbitrary spin S. The critical transverse field $\Gamma_c(T=0) = (z-1)SJ$, and the critical temperature $T_c(\Gamma=0) = \frac{1}{3}S(S+1)(z-1)J$ below which the ferromagnetic phase occurs, are obtained by using (7). In the classical limit $S \to \infty$, the partition function in (3) is replaced by

$$Z_i(\beta, \Gamma, H^{ef}) = \frac{\sinh\ \beta S\sqrt{\Gamma^2 + (H^{ef})^2}}{\beta S\sqrt{\Gamma^2 + (H^{ef})^2}}. \tag{8}$$

Then equation (7) gives the following transition lines:

$$(S/\Gamma)\coth\ \beta S\Gamma - 1/\beta\Gamma^2 = 1/[(z-1)J]. \tag{9}$$

The estimate of the ferromagnetic transition temperature $T_c(S)$ for $\Gamma = 0$ and $S \geqslant 1$ is

$$T_c(S) = \tfrac{1}{3}S^2(z-1)J. \tag{10}$$

On the other hand, it is well known that when we use the infinite-range interaction model given by

$$\tilde{J} = J/N \qquad z = N$$

the mean-field result can be obtained in the thermodynamic limit ($N \to \infty$).

The numerical analysis of (7) has been performed for $S = 1$, $\frac{3}{2}$ and 2. In figure 1 the phase diagrams in the (T, Γ) plane are shown. We see that the regions of ferromagnetic phase increase in size with increasing values of S, and the phase diagram in the spin-S case is similar to that obtained earlier for $S = \frac{1}{2}$.

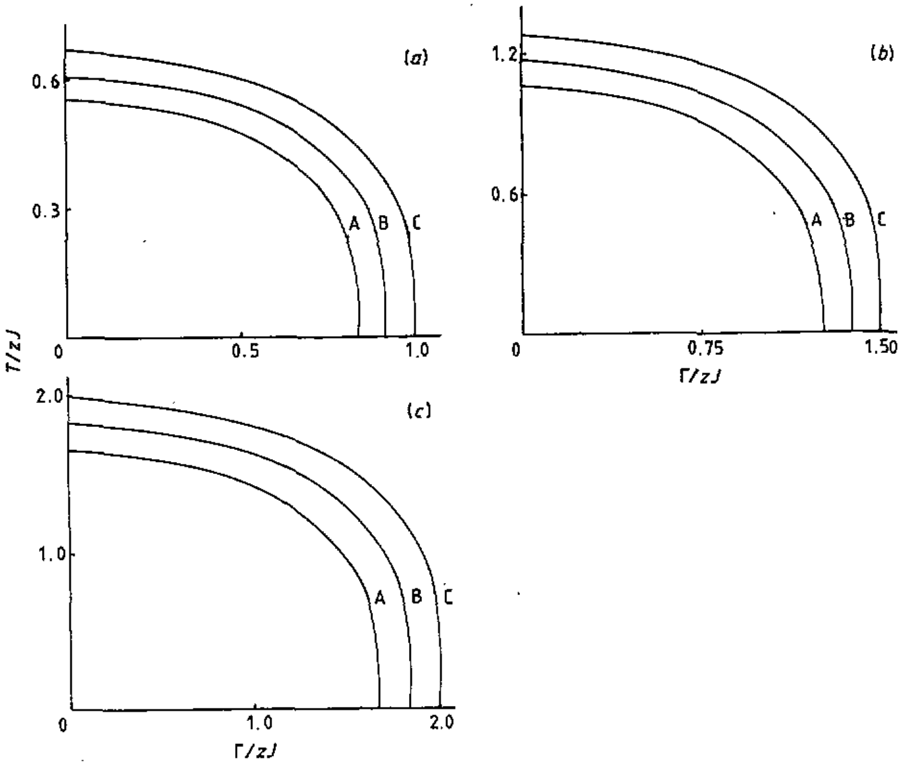

Figure 1. Phase diagram of the quantum transverse Ising model. Curve A, B and C correspond to $z = 6$, 12 and ∞, respectively. (*a*) $S = 1$; (*b*) $S = \frac{3}{2}$; (*c*) $S = 2$.

In conclusion, we have studied a transverse spin-S Ising model, using the pair approximation with the DPIR, which is superior to the MFA. The phase diagram has been obtained. Quantum fluctuations have the effect of destroying the ordered phase; the ferromagnetic phase disappears for $\Gamma \geqslant (z-1)SJ$. The results for the infinite-range interaction model can be recovered, and the classical-limit ($S \to \infty$) results are also discussed. It is easy to generalize the present approach to any mixed-spin Ising models in a transverse field.

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